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Evaluation of Two Eastern White Pine Site Index Equations at Biltmore Estate, North Carolina

W. Henry McNab, Bernard R. Parresol,
and Brian A. Ritter

The Authors

W. Henry McNab is a research forester with the Ecology and Management of Southern Appalachian Hardwoods Research Work Unit, U.S. Department of Agriculture, Forest Service, Southern Research Station, Asheville, **NC 28806; Bernard R Parresol** is a mathematical statistician with the Southern Forest Inventory, Monitoring, and Analysis Program (SFIMAP), U.S. Department of Agriculture, Forest Service, Southern Research Station, Asheville, NC 28804; and **Brian A.** Bitter is the forestry supervisor with the Biltmore Estate, Asheville, NC 28801.

Cover: Eastern white pines 106 years from seed on plot la of the Old Orchard Plantation, Biltmore Estate, in August 2000. Individually identified trees on this and two other study plots have been measured periodically for diameter andheight since October 1928, at 34 years.

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Asheville, NC 28802

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Abstract

The accuracy of two white pine (*Pinus strobus* L.) polymorphic site index equations was compared with field data from three plots in a loo-year-old stand at Biltmore Estate, Asheville, NC. One equation was developed from New Hampshire data and the other from Southern Appalachian data. Tree height has been measured periodically on those plots between stand age 34 and 72 years. At 50 years, site index ranged from 54 to 73 feet (ft). Absolute deviations of actual from estimated site index averaged 2.5 A for the Southern Appalachian equation and 2.7 ft for the New Hampshire equation. Site index estimates of the New Hampshire model were highly correlated (r = 0.92) with estimates of the Southern Appalachian model. Our evaluation indicates that the New Hampshire equation, which has desirable properties lacking in the Southern Appalachian equation, may be applicable on low- to moderate-quality sites beyond the area in which it was to be used. We suggest that a regional approach to site index development for white pine may be appropriate.

Keywords: Base-age invariant, height growth, invertible, *Pinus strobus* L., polymorphic, Southern Appalachian Mountains.

Introduction

Eastern white pine (*Pinus strobus* L.) is a widespread commercial timber species in the Eastern United States that responds well to management in planted and natural, as well as pure and mixed species stands, over a broad range of site quality (Burns and Honkala 1990). Methods for evaluating its site quality have long been available (Frothingham 1914), particularly based on site index (the height reached by a forest stand at a selected in&x age, usually 50 years)(Beck 197 lb). Parresol and Vissage (1998) developed a site index equation for eastern white pine (hereinafter white pine) in the Southern United States that has desirable characteristics lacking in other models. However, it was developed with data only from New Hampshire and was not evaluated for area of applicability. Although a polymorphic site index model is available for white pine in the Southern Appalachian Mountains (Beck 197 1 b), it is not as versatile as the New Hampshire model, nor has it been field tested using an independent data set.

Results of a study comparing predicted with actual white pine site index over a range of stand ages in the Southern Appalachian Mountains are presented in this paper. Its objectives are to compare precision of white pine site index estimates using two models (Beck 197 1 b, Parresol and Vissage 1998) and to compare site index estimates using the New Hampshire model with estimates using the Southern Appalachian model. The site index model developed by Beck (197 1 b) has long been accepted as applicable throughout the Southern Appalachian region and is used as a standard for comparison of other models.

Methods

Site Index Equations

Characteristics of the two site index equations are listed in table 1. Both are polymorphic, which permits more accurate modeling of height growth over a wide range of site qualities than do anamorphic equations. Invertibility facilitates estimation of stand height by rearrangement of the site index model form. Age invariance refers to changing the equation's index age without refitting the data. The Beck (1971 b) equation is expressed only in terms of height:

$$H = [63.06 + 0.67(S)]\{1-\exp^{-[0.00985+0.00033(S)]t}\}^2,$$

where

H = total mean dominant height (feet),
S = site index at base age 50 years (feet), and
t = stand age (years).

Estimates of site index must be obtained iteratively by varying stand height and age.

The Parresol and Vissage (1998) equation for base age 50 is formulated as

$$\ln S = 1/1.881 \exp(-8.6188/A)(\ln H + 74.7099/A - 2.0862) + 0.5920$$

where

ln = natural logarithm, and
A = stand age.

Table l-Characteristics of two equations developed for estimating site index of eastern white pine (*Pinus strobus* L.) in the Southern Appalachian Mountains

Characteristics of site index data and equation	**Beck (1971b)**	Parresol and Vissage (1998)
Data source	GA, NC, TN, VA	NH
Latitude range (degrees)	34-3 7°	43–45°
Longitude range (degrees)	82-84”	71-72”
Stands sampled (number)	42	196”
Age range (years)”	44–70	10-100
Stand height (feet)	n/a[c]	4-113
Site index range (50 yrs)	71-122	50-90
Polymorphic	Yes	Yes
Age invariant	No	Yes
Invertible	No	Yes

[a] **Summarized data from Frothingham (1914)**
[b] **Site index equation applicable from ages 5 to 70 years (Beck 1971b)**
[c] **Not available**

The Southern Appalachian equation was developed over a broader range of site indices than the New Hampshire equation, but the Parresol and Vissage (1998) equation is applicable in stands older than 70 years. Both models are applicable in stands as young as 10 years.

Study Area and Data

We conducted the field portion of this study in the Old Orchard Plantation on Biltmore Estate located near Asheville, North Carolina. This plantation was established with 4-year-old white pine seedlings planted at an average spacing of 3 by 4 ft in March 1899. In the fall of 1916, when the trees were 22 years from seed, a study was begun to investigate the effect of stand density on growth and yield. Three small contiguous plots: la (0.25 acre), lb (0.125 acre), and lc (0.125 acre), each surrounded by a 0.5-chain buffer zone, were established in a relatively uniform, 1.6-acre portion of the 5.6-acre stand (Haasis 1930). These plots have been the basis of periodic reports on diameter and volume increment, most recently by Della-Bianca (198 1). Trees on each plot were identified in 1916 and inventoried periodically for diameter at breast height (d.b.h.) and total height at ages 34, 41, 47, 52, 58, 72, and 100 years. We excluded the 100-year data from our tests because it exceeded the age range of the Southern Appalachian equation. Stand ages 34 and 4 1 were included, however, because the Southern Appalachian model is applicable at younger ages (Beck 197 1 b).

We selected sample trees using a method designed by Zeide and Zakrzewski (1993) in which a fixed number and a fixed proportion are chosen. The final number of sample trees ranged from 11 to 22 per plot, equivalent to between 88 and 96 trees per acre. We included only those trees alive at the last inventory so that crown class could be verified in the field as dominant or codominant.

Analysis

Heights of 11 and 16 sample trees were missing at ages 34 and 72 years, respectively; these were estimated using 4-point Lagrangian interpolation. The Lagrange polynomial through 4 points is

$$Y = Y_0 \frac{(X - X_1)(X - X_2)(X - X_3)}{(X_0 - X_1)(X_0 - X_2)(X_0 - X_3)} + Y_1 \frac{(X - X_0)(X - X_2)(X - X_3)}{(X_1 - X_0)(X_1 - X_2)(X_1 - X_3)} +$$

$$Y_2 \frac{(X - X_0)(X - X_1)(X - X_3)}{(X_2 - X_0)(X_2 - X_1)(X_2 - X_3)} + Y_3 \frac{(X - X_0)(X - X_1)(X - X_2)}{(X_3 - X_0)(X_3 - X_1)(X_3 - X_2)}$$

where

Y = interpolated tree height,
X = age at which Y is to be interpolated, and
Y_0, Y,, Y_2, Y_3 = respective heights at ages X_0, X,, X_2, X_3

For details on Lagrangian interpolation see Pachner (1984). Lagrangian interpolation was also used to estimate tree height at 50 years for determination of measured site index. We used the 95-percent confidence limits of measured site indices to assess the estimated values the two equations produced With field data from each of the six inventories, we used each equation to calculate site index. We then determined site index deviations (actual - estimated) at each periodic inventory for each equation. Predicted site index was then compared with the actual value determined at 50 years. Precision of the two equations was evaluated with percent standard error (Schlaegell98 1), using differences of actual - estimated site index at each of the six periodic inventories. The percent standard error is calculated as

$$S(\%) = \frac{\text{actual site index} - \text{estimated site index}}{\text{estimated site index}} \text{x}100.$$

This statistic indicates the size of error as a percentage of the mean of the site index distribution. Plotting and regression were used to further evaluate the relationship between site index predicted by the two equations. We used simple regression analysis to compare the relationship of site index estimated with the New Hampshire model to site index estimated with the Southern Appalachian model. In addressing the precision of his site index curves, Beck (197 1 b) wrote: "There was no evidence of bias at any age, and the accuracy expected in even the lo-year-old stands is probably **sufficient** for most management purposes."

Results and Discussion

Height growth of trees on all plots in the Old Orchard Plantation was nearly linear between ages 34 and 72 years (fig. 1). Annual increment ranged **from** 1 .O **ft** on plot 1 c to 1.5 ft on plots la and lb. The overall average of 1.38 **ft** is slightly greater than reported by Beck (197 la) for trees on **low-**quality sites in the Southern Appalachians. At 50 years, total heights of sample trees on the three plots averaged 53.8, **66.1, and 73.4 ft.**

We rounded mean sample tree height on each plot to the nearest foot to obtain site index; e.g., **54, 66,** and 73 **ft** (table 2). These values closely agree with previous estimates of 56, 7 1, and 74 **ft** that were calculated from the tallest seven trees per **0.125-acre** plot (Della-Bianca 1981). The standard deviation of heights at age 50 was greatest (7.8 **ft)** in plot la, which is situated between plots 1 b and 1 c and is more variable in site quality. These site in&x values are within the range of curves presented by Parresol and Vissage **(1998),** but not for the Southern Appalachian equation, which is limited to a minimum site index of 60 **ft.** To facilitate comparisons, we extrapolated Beck's (197 1 b) equation to site index 54.

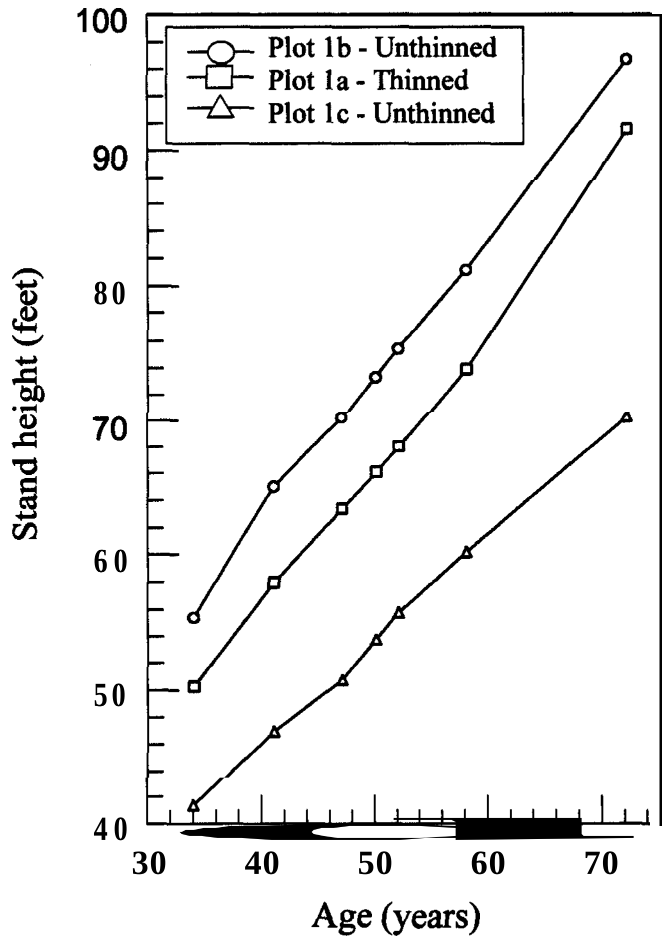

Figure l-Total height and age relationships of eastern white pines on three plots in the Old Orchard Plantation, Biltmore Estate, Asheville, NC. Plotting symbols indicate average height of the dominant and codominant stand at inventory years, except at age 50 years when height was interpolated.

Table 2—Characteristics of dominant and codominant eastern white pine stands interpolated at 50 years of age for determination of site index on three plots in the Old Orchard Plantation, Biltmore Estate, Asheville, NC

statistic	Plot lc	Plot la	Plot 1 b
Plot area (acre)	0.125	0.25	0.125
Trees sampled (number)	12	22	11
Age **from** seed (yrs)	50	50	50
Mean stand height **(ft)**[a]	53.8	66.1	73.4
Minimum tree height **(ft)**[a]	47.8	47.7	66.9
Maximum tree height **(ft)**[a]	62.4	78.0	81.7
Standard deviation **(ft)**	4.8	7.8	4.6
Standard error **(ft)**	1.4	1.6	1.4

[a] Determined by **Lagrangian** interpolation

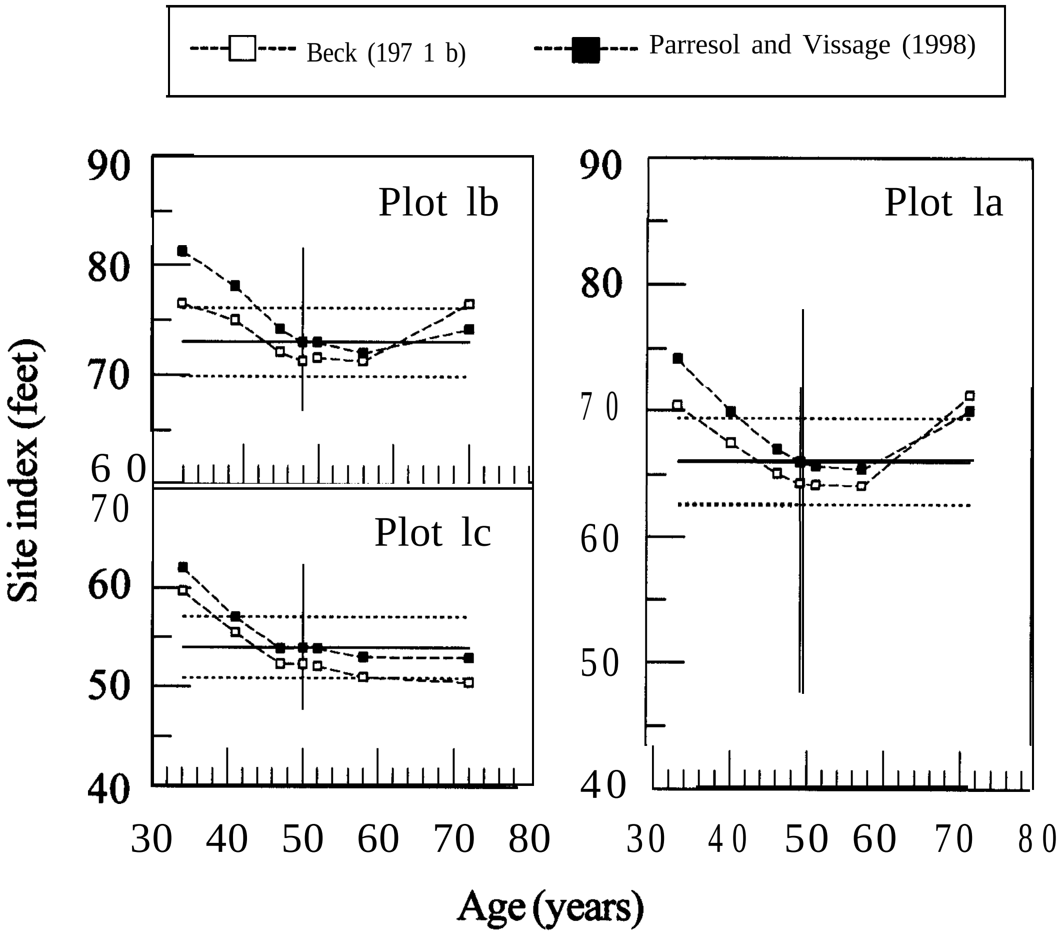

Figure 2-Precision of white pine site index estimation at periodic inventories using equations developed by Beck (1971b) and Parresol and Vissage (1998) on three sample plots in the Old Orchard Plantation, Biltmore Estate, Asheville, NC. The range of total heights of sample trees interpolated at 50 years of age on each plot is indicated by length of the vertical line. The two parallel, horizontal, dashed lines delineate the 95-percent confidence interval of mean site index.

The pattern of estimated to actual site index was consistent for both equations on all plots (fig. 2). Site index estimates of both equations were within the 95-percent confidence limits between years 40 and about 65. The Southern Appalachian model was particularly ill-behaved beyond age 60 when estimates of site index abruptly increased and crossed estimates of the New Hampshire equation. Mean deviations of site index were greatest (2.8 ft) in plot lc, and deviations were least (2.2 ft) in plot lb (table 3). For all plots combined, mean absolute deviations of site index were about equal for the Southern Appalachian equation (2.53 ft) and the New 'Hampshire equation (2.67 ft). One difference between the two equations is that stand height is equivalent to site index at the base age of 50 years for the New Hampshire equation but not for the Southern Appalachian equation. Estimated site index based on Beck's (197 1 b) model averages 2.7 percent less than tree height at the base age.

Comparison of standard errors for stand ages 34 to 72 years indicates no clear trend of site index estimates between the two equations (table 3). Maximum standard errors occurred at the youngest plantation age (34 years) for the New Hampshire equation but at the oldest (72 years) for the Southern Appalachian equation. Errors were smallest for plot lb in both equations. For young stand ages (30 to 40 years), where estimates of site index are important for estimating future yield, Beck's (1971b) equation is most accurate, although at 34 years neither equation provided satisfactory results. For estimates beyond 40 years, errors were smallest with the New Hampshire equation. Users should remember that our tests were conducted in three small plots in a single

Table 3-Average total height, deviation, and percent standard error of two site index equations applied at periodic inventories in three plots of the Old Orchard White Pine Plantation, Biltmore Estate, Asheville, NC

Plot numbers	Age	Height	Beck (1971b) (Dev.)	Beck (1971b) S(%)[a]	Parresol and Vissage (1998) (Dev.)	Parresol and Vissage (1998) S(%)
	Yrs	*Ft*				
lc	34	41.4	-5.7	-9.5	-8.0	-12.9
lc	41	47.0	-1.5	-2.7	-3.1	-5.4
lc	47	50.8	1.7	3.3	0.1	0.1
lc	52	55.8	1.9	3.6	0.1	0.2
lc	58	60.2	3.0	5.9	1.0	1.8
lc	72	70.3	3.0	-5.6	1.0	2.0
Mean			0.4	-0.8	-1.5	-2.4
Absolute mean			2.8	5.1	2.2	3.7
la	34	50.3	-4.4	-6.3	-8.2	-11.1
la	41	58.0	-1.5	-2.2	-3.9	-5.6
la	47	63.4	0.9	1.4	-1.0	-1.5
la	52	68.0	1.8	2.8	0.3	0.4
la	58	73.9	1.9	3.0	0.6	0.9
la	72	91.6	-5.2	-11.1	-4.0	-5.7
Mean			-1.1	-2.1	-2.7	-3.8
Absolute mean			2.6	4.5	3.0	4.2
lb	34	55.5	-3.5	-9.0	-8.3	-10.2
lb	41	65.1	-2.0	-5.3	-5.1	-6.6
lb	47	70.3	0.9	-0.3	-1.2	-1.6
lb	52	75.5	1.4	1.3	0.0	-0.1
lb	58	81.2	1.7	2.7	0.9	1.3
lb	72	96.8	-3.5	-0.3	-1.2	-1.6
Mean			-0.8	-1.8	-2.5	-3.1
Absolute mean			2.2	3.2	2.8	3.6
Overall mean			-0.50	-1.57	-2.22	-3.09
Absolute overall mean			2.53	4.24	2.67	3.83

[a] S(%) = [(actual - estimated)/estimated] 100

stand on a site of lower productivity **than** is **typical** of the Southern Appalachians. Our results should not be extrapolated to other areas without first conducting additional validation tests.

Site in&x estimated using the New Hampshire model was highly correlated (**r^2** = 0.92) with the Southern Appalachian model (fig. 3). We observed that the greatest deviation tended to be associated with site index estimates at the youngest and oldest ages. According to our regression analysis, the New Hampshire model expresses a relationship similar to that of the Southern Appalachian model between site indices 5 1 to 76 (regression slope = **0.98),** but estimates of the New Hampshire model slightly exceed those of the Southern Appalachian model (intercept = 2.1 **ft).** The close agreement of site index estimates between the two models suggests that development of a single regional equation for white pine may be feasible.

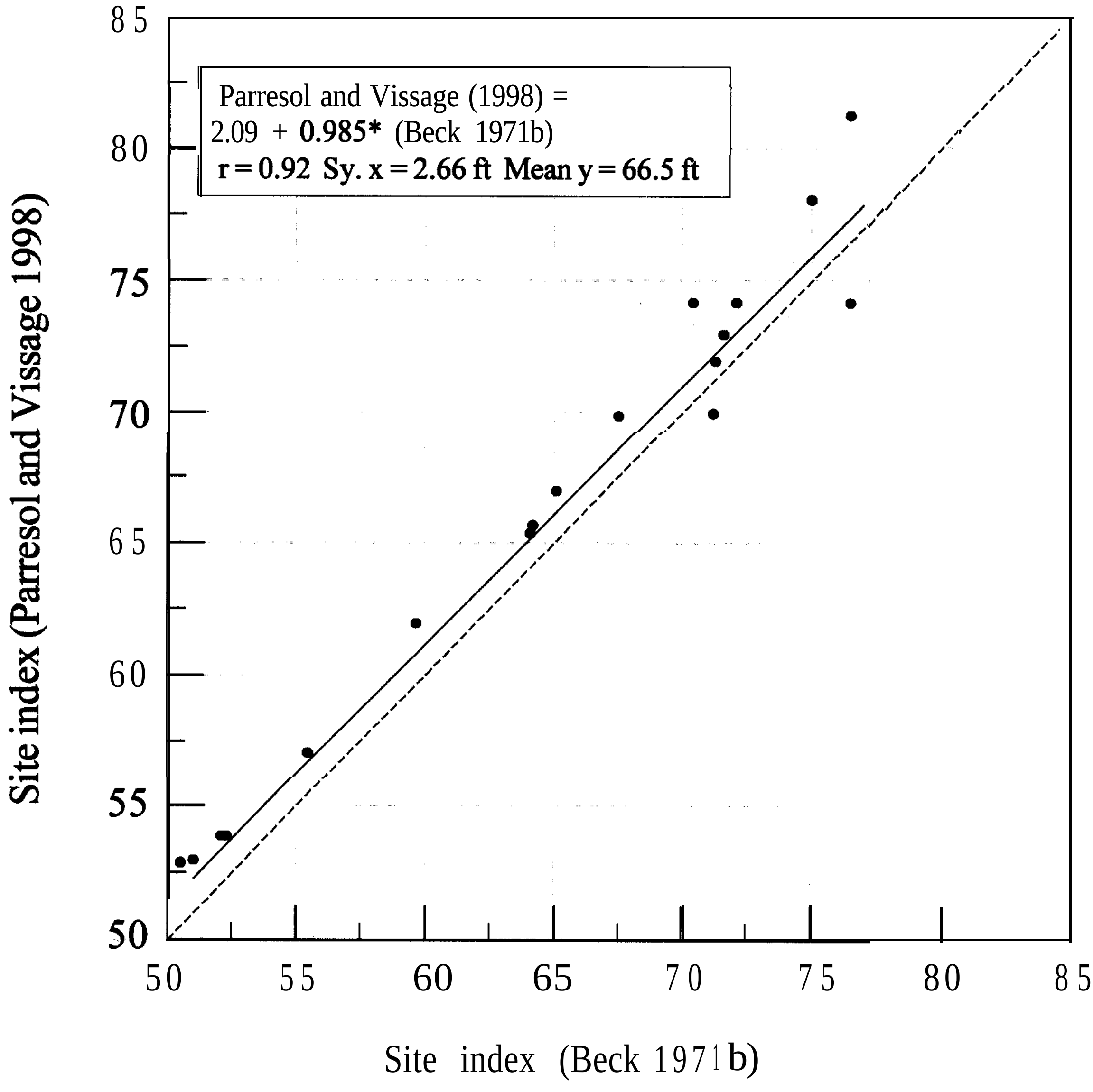

Figure 3-Site index estimates of eastern white pine determined by Beck (1971b) and **Parresol** and Vissage (1998) in the Old Orchard Plantation, Biltmore Estate, Asheville, NC. The dashed line indicates perfect correlation between the two models. The solid line represents the least squares linear regression for the New Hampshire model as a function of site index estimated by the accepted standard model (Beck 1971b).

Several explanations may be relevant for the results of this study. First, tree height growth does not exhibit the typical sigmoid shape assumed by the equations. The height increment of white pine stands on low-quality sites such as the Old Orchard is typically linear (Beck 197 1 a); but neither site index equation adequately described relationships among the three plots. Second, both equations were developed with data from natural stands but were applied in a stand that had been planted and a portion of which had been thinned. The effects of vegetation control on early height growth during white pine stand development are unknown. Third the effects of thinning (plot 1 a) on white pine height increment are unknown. Lastly, at very young and old stand ages, the different equation formulations used in the two models likely account for some **of the** lack of agreement between the two site index equations. These variable vegetative responses may cause users to question site index as an ideal measure of site quality (Beck and Trousdell973).

In summary, the two equations performed similarly on three relatively low-quality sites in one stand in the Southern Appalachian Mountains. However, neither accurately depicts the linear pattern of white pine height increment on the small and unreplicated plots of the Old Orchard stand. Estimates of actual site index were slightly more accurate using the Southern Appalachian equation for stand ages less than 40, but they were more accurate using the New Hampshire equation in stands 40 years and older. We suggest that it is desirable and feasible to develop a regional site index equation for white pine.

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